Estimating the Mean

In section 7.3 the finite population correction (fpc) factor is used to reduce the standard error by a value equal to $\sqrt{(N-n)/(N-1)}$. When developing confidence interval estimates for population parameters, the fpc factor is used when samples are selected without replacement. Thus, the $(1 - \alpha) \times 100\%$ confidence interval estimate for the mean is calculated as in Equation (8.12).

$$\bar{X} \pm t_{n-1} \frac{S}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$$

(8.12)

To illustrate the finite population correction factor, refer to the confidence interval estimate for the mean developed for Saxon Plumbing Company in section 8.2. Suppose that in this month there are 5,000 sales invoices. Using $\bar{X} = \$110.27$, $S = \$28.95$, $N = 5,000$, $n = 100$, and with 95% confidence, $t_{99} = 1.9842$. From equation (8.12)

$$110.27 \pm (1.9842) \frac{28.95}{\sqrt{100}} \sqrt{\frac{5,000 - 100}{5,000 - 1}}$$

$$= 110.27 \pm 5.744(0.99)$$

$$110.27 \pm 5.69$$

$\$104.58 \leq \mu \leq \$115.96$

Figure 8.25 illustrates the PHStat output for this example.

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**Figure 8.25**
Confidence interval estimate for the mean sales invoice amount with the finite population correction for Saxon Plumbing Company

In this case, because the sample is a very small fraction of the population, the correction factor has a minimal effect on the width of the confidence interval. To examine the effect of the correction factor when the sample size is more than 5% of the population size, see Example 8.10.
In Example 8.3, a sample of 35 single-family homes was selected. Suppose a population of 500 single-family homes is served by the company. Set up a 95% confidence interval estimate of the population mean.

**SOLUTION** Using the finite population correction factor, with $\bar{X} = 1,122.7$ gallons, $S = 295.72$, $n = 35$, $N = 500$, and $t_{34} = 2.0322$ (for 95% confidence):

$$\bar{X} \pm t_{n-1} \frac{S}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$$

$$= 1,122.7 \pm (2.0322) \frac{295.72}{\sqrt{35}} \sqrt{\frac{500-35}{500-1}}$$

$$= 1,122.7 \pm 101.58(0.9653)$$

$$= 1,122.7 \pm 98.06$$

$$1,024.64 \leq \mu \leq 1,220.76$$

Here, because more than 5% of the population is to be sampled, the fpc factor has a moderate effect on the confidence interval estimate.

**Estimating the Proportion**

In sampling without replacement, the $(1 - \alpha) \times 100\%$ confidence interval estimate of the proportion is found as shown in Equation (8.13)

$$p_s \pm Z \sqrt{\frac{p_s(1-p_s)}{n}} \sqrt{\frac{N-n}{N-1}}$$ (8.13)

To illustrate the use of the finite population correction factor when developing a confidence interval estimate of the population proportion, consider again the estimate developed for Saxon Plumbing Company in section 8.3. For these data, $N = 5,000$, $n = 100$, $p_s = 10/100 = 0.10$, and with 95% confidence, $Z = 1.96$. Using Equation (8.13),

$$p_s \pm Z \sqrt{\frac{p_s(1-p_s)}{n}} \sqrt{\frac{N-n}{N-1}}$$

$$= 0.10 \pm (1.96) \sqrt{\frac{(0.10)(0.90)}{100}} \sqrt{\frac{5,000-100}{5,000-1}}$$

$$= 0.10 \pm (1.96)(0.03)(0.99)$$

$$= 0.10 \pm 0.0582$$

$$0.0418 \leq p \leq 0.1582$$

In this case, because the sample is a very small fraction of the population, the fpc factor has virtually no effect on the confidence interval estimate.
Determining the Sample Size

Just as the fpc factor is used to develop confidence interval estimates, it also is used to determine sample size when sampling without replacement. For example, in estimating the mean, the sampling error is

\[ e = \frac{Z \sigma}{\sqrt{n}} \sqrt{\frac{N - n}{N - 1}} \]

and in estimating the proportion, the sampling error is

\[ e = \frac{Z \sqrt{p(1-p)}}{\sqrt{n}} \sqrt{\frac{N - n}{N - 1}} \]

To determine the sample size in estimating the mean or the proportion from Equations (8.4) and (8.5)

\[ n_0 = \frac{Z^2 \sigma^2}{e^2} \quad \text{and} \quad n_0 = \frac{Z^2 p(1-p)}{e^2} \]

where \( n_0 \) is the sample size without considering the finite population correction factor. Applying the fpc factor to this results in the actual sample size \( n \), computed as in Equation (8.14).

\[
\text{SAMPLE SIZE DETERMINATION USING THE FINITE POPULATION CORRECTION FACTOR}
\]

\[ n = \frac{n_0 N}{n_0 + (N - 1)} \quad (8.14) \]

In the determining sample size for Saxon Plumbing Company, a sample size of 97 was needed for the mean and a sample of 100 was needed for the proportion. Using the fpc factor in Equation (8.14) for the mean, with \( N = 5,000 \), \( e = 5 \), \( S = 25 \), and \( Z = 1.96 \) (for 95% confidence), leads to

\[ n = \frac{(96.04)(5,000)}{96.04 + (5,000 - 1)} = 94.24 \]

Thus, \( n = 95 \).

Using the fpc factor in Equation (8.14) for the proportion, with \( N = 5,000 \), \( e = 0.07 \), \( p = 0.15 \), and \( Z = 1.96 \) (for 95% confidence),

\[ n = \frac{(99.96)(5,000)}{99.96 + (5,000 - 1)} = 98.02 \]

Thus, \( n = 99 \).
To satisfy both requirements simultaneously with one sample, the larger sample size of 99 is needed. PHStat output is displayed in Figure 8.26.

![Figure 8.26](image)

**PROBLEMS FOR SECTION 8.7**

**Learning the Basics**

8.86 If $X = 75, S = 24, n = 36,$ and $N = 200,$ set up a 95% confidence interval estimate of the population mean $\mu$ if sampling is done without replacement.

8.87 Consider a population of 1,000 where the standard deviation is assumed equal to 20. What sample size would be required if sampling is done without replacement if you desire 95% confidence and a sampling error of ±5?

**Applying the Concepts**

8.88 The quality control manager at a lightbulb factory needs to estimate the mean life of a large shipment of lightbulbs. The process standard deviation is known to be 100 hours. Assume that the shipment contains a total of 2,000 lightbulbs and that sampling is done without replacement.

a. set up a 95% confidence interval estimate of the population mean life of light bulbs in this shipment if a random sample of 50 lightbulbs selected from the shipment indicates a sample average life of 350 hours.

b. determine the sample size needed to estimate the average life to within ±20 hours with 95% confidence.

c. What are your answers to (a) and (b) if the shipment contains 1,000 lightbulbs?

8.89 A survey is planned to determine the mean annual family medical expenses of employees of a large company. The management of the company wishes to be 95% confident that the sample average is correct to within ±$50 of the true average annual family medical expenses. A pilot study indicates that the standard deviation is estimated as $400. How large a sample size is necessary if the company has 3,000 employees and if sampling is done without replacement?

8.90 The manager of a bank that has 1,000 depositors in a small city wants to determine the proportion of its depositors with more than one account at the bank.

a. Set up a 90% confidence interval estimate of the population proportion of the bank’s depositors who have more than one account at the bank if a random sample of 100 depositors is selected without replacement and 30 state that they have more than one account at the bank.

b. A bank manager wants to be 90% confident of being correct to within ±0.05 of the true population proportion of depositors who have more than one account at the bank. What sample size is needed if sampling is done without replacement?

c. What are your answers to (a) and (b) if the bank has 2,000 depositors?

8.91 An automobile dealer wants to estimate the proportion of customers who still own the cars they purchased 5 years earlier. Sales records indicate that the population of owners is 4,000.

a. Set up a 95% confidence interval estimate of the population proportion of all customers who still own their cars 5 years after they were purchased if a random sample of 200 customers selected without replacement from the automobile dealer’s records indicate that 82 still own cars that were purchased 5 years earlier.

b. What sample size is necessary to estimate the true proportion to within ±0.025 with 95% confidence?

c. What are your answers to (a) and (b) if the population consists of 6,000 owners?
8.92 The inspection division of the Lee County Weights and Measures Department is interested in estimating the actual amount of soft drink that is placed in 2-liter bottles at the local bottling plant of a large nationally known soft-drink company. The population consists of 2,000 bottles. The bottling plant has informed the inspection division that the standard deviation for 2-liter bottles is 0.05 liter.

a. Set up a 95% confidence interval estimate of the population mean amount of soft drink in each bottle if a random sample of one hundred 2-liter bottles obtained without replacement from this bottling plant indicates a sample average of 1.99 liters.

b. Determine the sample size necessary to estimate the population mean amount to within ±0.01 liter with 95% confidence.

c. What are your answers to (a) and (b) if the population consists of 1,000 bottles?

8.93 A stationery store wants to estimate the mean retail value of greeting cards that it has in its 300-card inventory.

a. Set up a 95% confidence interval estimate of the population mean value of all greeting cards that are in its inventory if a random sample of 20 greeting cards selected without replacement indicates an average value of $1.67 and a standard deviation of $0.32.

b. What is your answer to (a) if the store has 500 greeting cards in its inventory?