How fast must an object move before its length appears to be contracted to \( \frac{3}{4} \) its proper length?

\[
L = L_0 \sqrt{1 - \beta^2} \implies \frac{L}{L_0} = \sqrt{1 - \beta^2} = \frac{3}{4} = 0.75
\]

\[
1 - \beta^2 = 0.526 \implies \beta^2 = 0.4375
\]

\[
\beta = 0.66 \implies v = 0.66c
\]

High-energy particles are observed in laboratories by photographing the tracks they leave in certain detectors (For example, a bubble chamber pictures, see the “notes” page for a description of how to read one.); the length of the track depends on the speed of the particle and its lifetime. A particle moving at 0.990c leaves a track 1.00 mm long. What is the proper lifetime of the particle?

I, in the laboratory rest frame, measure the track to be 1.00 mm long. Thus, I measure it’s lifetime to be:

\[
t = \frac{x}{v} = \frac{0.001}{0.990c} = 3.37 \times 10^{-12} s
\]

But this is not the proper time. It is the dilated time. The proper time is measured by the particle (where it remains at the same location in it’s rest frame.) Thus, using the time dilation equation, and solving for the proper lifetime:

\[
T = \frac{T_0}{\sqrt{1 - \beta^2}} \implies T_0 = T \sqrt{1 - \beta^2} = \left(3.37 \times 10^{-12}\right) \sqrt{1 - 0.99^2} s = 4.75 \times 10^{-13} s
\]

So, I see the particle for a much longer time than I would if it were sitting still (relative to me.) As I look at the particle, and perceive it as moving, I note that it’s “clock” (the behavior that would make it decay within 0.475 ps) is running slow... and it “lives” longer!

Another way to approach this same problem is to note that the distance I measure (1.00 mm) is in the lab. According to the particle, this distance will be contracted in it’s frame of reference.

\[
L = L_0 \sqrt{1 - \beta^2} = (1.00) \sqrt{1 - 0.99^2} mm = 0.14 mm
\]

So, in it’s frame of reference, it goes 0.14 mm, while traveling at 0.99c:

\[
t = \frac{x}{v} = \frac{0.14}{0.99c} = 4.75 \times 10^{-13} s
\]

Note that we have consistency in the “event” that we observe (the decay of the particle).
Rocket A leaves a space station with a speed of 0.826c. Later, rocket B leaves in the same direction with a speed of 0.635c. What is the speed of rocket A as observed from rocket B?

The key in these problems is to properly define our variables:

The stationary reference frame (S) is the space station. We already know all speeds relative to this frame of reference. The moving frame of reference (S’) is rocket B. We know how it moves relative to S, and we wish to know the observed motion of rocket A in its frame of reference.

Now, before we jump in, let’s make a logical assessment of what observers in B will see. The observers in S see A moving at 0.826c. B moves in the same direction, but slower. So, although it is not “catching up”, B should see A moving relatively slower than space station people... right?.

\[
u_x' = \frac{u_x - v}{1 - (u_x v / c^2)} = \frac{0.826c - 0.635c}{1 - (0.826)(0.635)} = \frac{0.191c}{0.47549} = 0.402c
\]

Several spacecraft leave a space station at the same time. Relative to an observer on the station, A travels at 0.60c in the x direction, B at 0.50c in the y direction, C at 0.50c in the negative x direction, and D at 0.50c at 45° between the y and negative x directions. Find the velocity components, directions, and speeds of B, C, and D as observed from A.

You are the observer in A, moving in the +x-direction at 0.60c, and we want to know how you observer all these motions. If we call your frame of reference the S’ frame, and we know all the motions in S, then we can use the Lorentz Velocity Transformations:

\[
u_x' = \frac{u_x - v}{1 - (u_x v / c^2)}; \quad u_y' = \frac{u_y \sqrt{1 - \beta^2}}{1 - (u_x v / c^2)}; \quad u_z' = \frac{u_z \sqrt{1 - \beta^2}}{1 - (u_x v / c^2)}
\]

B: For a spaceship traveling with \( u_B = 0.5c \hat{y} \)

\[
u_x' = \frac{0 - 0.6c}{1 - (0v/c^2)}; \quad u_y' = \frac{0.5c \sqrt{1 - 0.6^2}}{1 - (0v/c^2)}; \quad u_z' = \frac{0 \sqrt{1 - 0.6^2}}{1 - (0v/c^2)}
\]

\[
u_x' = -0.6c; \quad u_y' = 0.4c; \quad u_z' = 0
\]

\[
u_B' = \sqrt{0.6^2 + 0.4^2} c = 0.72c; \quad \theta_B = \tan^{-1} \left( \frac{0.4c}{-0.6c} \right) = 146°
\]
Electrons are accelerated to high speeds by a two-stage machine. The first stage accelerates the electrons from rest to \( v = 0.90c \). The second stage accelerates the electrons from \( 0.90c \) to \( 0.99c \).

(a) How much energy does the first stage add to the electrons?

(b) How much energy does the second stage add in increasing the velocity by only 10 percent?

Let’s find the total energy the electrons have after the first stage.

\[
E = mc^2 = \frac{m_0c^2}{\sqrt{1 - \beta^2}} = \frac{0.511\text{MeV}}{\sqrt{1 - (0.9)^2}} = 1.17\text{MeV}
\]

So, they gained \( 0.66\text{MeV} \) of KE. This is the WORK done by the machine.

Now, let’s see how much total energy the electrons have after the second stage:

\[
E = mc^2 = \frac{m_0c^2}{\sqrt{1 - \beta^2}} = \frac{0.511\text{MeV}}{\sqrt{1 - (0.99)^2}} = 3.62\text{MeV}
\]

So, the net gain here is only \( (3.62-1.17)\text{ MeV} = 2.45\text{ MeV} \). Again, this is the WORK done by the machine.

Note that the 2nd stage requires more than twice as much work, but only increases the speed by 0.09c or 10%!